한양대학교

답안지 (자연계열)

수 험 번 호	성 명	페 이 지

[문제 1-1]

$f(a\sin x_1) = f(a\sin x_2)$ 라 하자.

$$\frac{a^2\sin^2 x_1}{a\sin x_1 + \sqrt{a^2 - a^2\sin^2 x_1}} = \frac{a^2\sin^2 x_1}{a(\sin x_1 + \cos x_1)} = \frac{a\sin^2 x_1}{\sin x_1 + \cos x_1}$$

$$\frac{a^2\sin^2 x_2}{a\sin x_2 + \sqrt{a^2 - a^2\sin^2 x_2}} = \frac{a^2\sin^2 x_2}{a(\sin x_2 + \cos x_2)} = \frac{a\sin^2 x_2}{\sin x_2 + \cos x_2}$$

$$\frac{a\sin^2 x_1}{\sin x_1 + \cos x_1} = \frac{a\sin^2 x_2}{\sin x_2 + \cos x_2} \quad \cdots ①$$

$f(a\cos x_1) = f(a\cos x_2)$ 라 하자

$$\frac{a\cos^2 x_1}{\cos x_1 + \sin x_1} = \frac{a\cos^2 x_2}{\cos x_2 + \sin x_2} \quad \cdots ②$$

①+② 를 하자

$$\frac{1}{\cos x_1 + \sin x_1} = \frac{1}{\cos x_2 + \sin x_2}$$

$$\cos x_2 + \sin x_2 = \cos x_1 + \sin x_1$$

$$\frac{1}{\sqrt{2}}(\cos x_2 + \sin x_2) = \frac{1}{\sqrt{2}}(\cos x_1 + \sin x_1)$$

$$\sin\left(x_2 + \frac{\pi}{4}\right) = \sin\left(x_1 + \frac{\pi}{4}\right)$$

$$x_2 + \frac{\pi}{4} + 2n\pi = x_1 + \frac{\pi}{4}$$

$$x_2 + 2n\pi = x_1$$

$\sin x_1 = \sin(x_2 + 2n\pi) = \sin x_2$ 이고

$\cos x_1 = \cos(x_2 + 2n\pi) = \cos x_2$ 이므로

따라서 $f(x)$는 일대일 함수가 된다.

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문제 1-2]

$$\int_0^{\frac{\pi}{2}} x f(a\sin x)\,dx = \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right) f\left(a\sin\left(\frac{\pi}{2}-x\right)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right) f(a\cos x)\,dx$$

$$\int_{\frac{\pi}{2}}^{\pi} x f(a\sin x)\,dx = \int_0^{\frac{\pi}{2}} \left(x+\frac{\pi}{2}\right) f\left(a\sin\left(x+\frac{\pi}{2}\right)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(x+\frac{\pi}{2}\right) f(a\cos x)\,dx$$

$$\int_0^{\frac{\pi}{2}} x f(a\sin x)\,dx + \int_{\frac{\pi}{2}}^{\pi} x f(a\sin x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right) f(a\cos x)\,dx + \int_0^{\frac{\pi}{2}} \left(x+\frac{\pi}{2}\right) f(a\cos x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{2}} f(a\cos x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{2}} f\left(a\cos\left(\frac{\pi}{2}-x\right)\right) dx$$

$$= \pi \int_0^{\frac{\pi}{2}} f(a\sin x)\,dx$$

$$\therefore \int_0^{\pi} x f(a\sin x)\,dx = \int_0^{\frac{\pi}{2}} x f(a\sin x)\,dx + \int_{\frac{\pi}{2}}^{\pi} x f(a\sin x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{2}} f(a\sin x)\,dx$$ 이다.

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[문제 1-3]

$$\int_0^{\pi} x f(a\sin x)\,dx = \pi \int_0^{\frac{\pi}{2}} f(a\sin x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{a^2\sin^2 x}{a\sin x + \sqrt{a^2 - a^2\sin^2 x}}$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{a^2\sin^2 x}{a\sin x + a\cos x}$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{a\sin^2 x}{\sin x + \cos x}$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{a\cos^2 x}{\sin x + \cos x}$$ (x에서 $\frac{\pi}{2} - x$ 를 집어넣어주면 같아진다.)

$$\pi \int_0^{\frac{\pi}{2}} \frac{a}{\sin x + \cos x} = \frac{a}{\sqrt{2}}\pi \int_0^{\frac{\pi}{2}} \csc\left(x + \frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}a}{2}\pi \int_0^{\frac{\pi}{2}} \csc\left(x + \frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}a}{2}\pi \left[-\ln\left(\cot\left(x + \frac{\pi}{4}\right) + \csc\left(x + \frac{\pi}{4}\right)\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\sqrt{2}a}{2}\pi \left\{-\ln\left(\cot\frac{3}{4}\pi + \csc\frac{3}{4}\pi\right) + \ln\left(\cot\frac{\pi}{4} + \csc\frac{\pi}{4}\right)\right\}$$

$$= \frac{\sqrt{2}a}{2}\pi \left(-\ln(\sqrt{2}-1) + \ln(\sqrt{2}+1)\right)$$

$$= \frac{\sqrt{2}a}{2}\pi \left(\ln(\sqrt{2}+1)^2\right)$$

$$= \sqrt{2}a\pi\left(\ln(\sqrt{2}+1)\right)$$

$$\therefore \int_0^{\pi} x f(a\sin x)\,dx = \frac{\sqrt{2}}{2}a\pi \ln(\sqrt{2}+1)$$